

Physics I

Set no. 1



Calculations and basic mathematics

E - exam type question / task

1. Please write the following numbers to the three significant digits:

- a) 12 443
- b) 134
- c) 120 013
- d) $6.432 \cdot 10^{12}$
- e) 0.003343
- f) 1.4922
- g) 2.40149
- h) 0.00420042
- i) $2.100242 \cdot 10^{-7}$

2. Please express in

- radians:

30 deg, 12 deg, 150 deg, 450 deg

- degrees:

$\pi/3$ rad ; $4\pi/5$ rad ; 1.57 rad ; 0.23 rad

3. Please calculate - if necessary round to the three significant digits (consider giving the result in exponential notation):

a) $(5.0 \cdot 10^4) \cdot (3.0 \cdot 10^6)$

b) $(5.0 \cdot 10^4) \cdot (3.0 \cdot 10^{-6})$

c) $5.0 \cdot 10^5 + 3.0 \cdot 10^6$

d) $(7.0 \cdot 10^6)/(2.0 \cdot 10^{-6})$

e)
$$\frac{\sqrt[3]{2.53 \cdot 10^5 - 1.1 \cdot 10^3} + \sqrt{\frac{1225}{2^8}} \cdot 10^2}{\sin(30^\circ) \cdot \ln(2^5 \cdot 5)}$$

f)
$$\frac{\cos\left(\frac{\sqrt{12.56}}{15}\right) + \sin\left(\frac{\pi}{5}\right)}{\arcsin(0.5) - 0.97^3}$$

4. Please find the number of significant figures in the following numbers:

- a) 0.003352
- b) 0.043034245
- c) 2.032
- d) 12.43
- e) 12.430
- f) 12.4300
- g) 0.0043200300

5. Please solve the formulas to determine the given quantity (letter):

a) $ma = F - mgf \cos \alpha \Rightarrow m = ?$

b) $\frac{GMm}{r^2} = \frac{m(\frac{2\pi}{T})^2}{r} \Rightarrow r = ?$

c) $v = \sqrt{v_0^2 + 2gh} \Rightarrow h = ?$

6. Please find a and t as a function of v_0 and s if:

$$\begin{cases} s = v_0 t + \frac{1}{2} at^2 \\ v = v_0 + at \end{cases}$$

7. Please solve the equations (x – a real number):

a) $2x^2 - 5x + 3 = 0$

b) $(x - 4)^2 = 16$

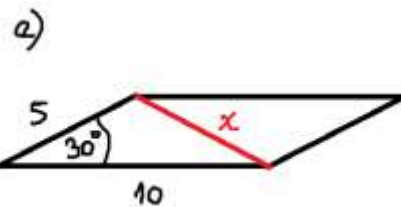
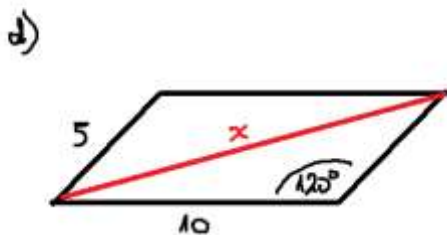
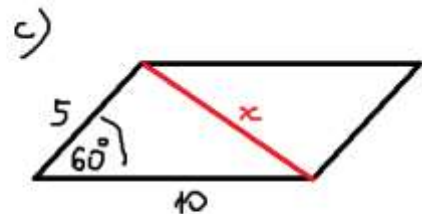
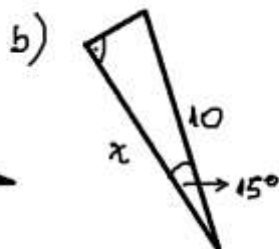
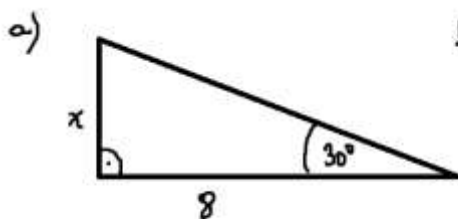
c) $\sqrt{2(x-5)} = x-17$

d) $x^3 - 5x^2 - 2x + 10 = 0$

e) $2x - 1 = \frac{x-1}{0.5x - \frac{4}{7}}$

f) $2A \cos(0.5x) = A, x \in [0, 90^\circ]$

8. Please find x if:



c), d), e) – parallelograms

9. Let us assume: a, b – real numbers; $a * b = \begin{cases} |a - b|, & \text{if } a > 0, b > 0 \\ 2a - b, & \text{if } ab < 0 \\ 15, & \text{if } a = b, a < 0 \\ a^2 - b, & \text{in the rest} \end{cases}$

Please determine:

- a) 2^3
- b) $(-3)^5$
- c) $0^{(-1)}$
- d) $x^x, x < 0$
- e) $x^x, x - \text{any real number}$

Units, SI units

10. Please fill the gaps (consider exponential notation):

- a) $6.23 \text{ mm} = \dots\dots\dots \text{ m} = \dots\dots\dots \text{ km}$
- b) $2.32 \text{ ms} = \dots\dots\dots \text{ s}$
- c) $0.0324 \text{ } \mu\text{g} = \dots\dots\dots \text{ kg}$
- d) $18 \text{ km/h} = \dots\dots\dots \text{ m/s}$

11. Please find the result in SI units and name the physical quantity that could be the result:

- a)
$$\frac{6 \frac{\text{km}}{\text{h}} \cdot 15 \text{ min} + 8 \frac{\text{m}}{\text{s}} \cdot \frac{1}{6} \text{ h}}{2000 \text{ s}} =$$
- b)
$$\frac{2 \cdot 0.135 \text{ km}}{(0.015 \text{ h})^2} =$$
- c)
$$\frac{3000 \text{ g} \cdot 10 \frac{\text{m}}{\text{s}^2} + 150 \text{ mN} + \frac{12 \text{ J}}{0.002 \text{ km}}}{1350 \text{ cm}^2 + 0.16 \text{ m} \cdot 350 \text{ mm}} =$$



12. Suppose $A = BC$, where A has the dimension $[A] = L/M$ and C has the dimension $[C] = L/T$. Please find $[B]$ - the dimension of B .



13. Suppose $A = B^2 C^{-1/3}$. Please find $[A]$, if:

- a) $[B] = LM$; $[C] = L/T^2$
- b) $[B] = L/M^2$; $[C] = L^3 T^6$
- c) $[B] = M$; $[C] = T^2/L^3$



14. Suppose $A = B^n C^m$, where $[A] = LT$, $[B] = L^2 T^{-1}$, and $[C] = LT^2$. Please find values of m and n

15. Please find a volume and a total surface of (express in m^3 or m^2):

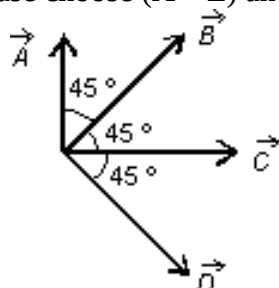
- a) a sphere with a radius of 1.7 cm,
- b) a cylinder with a radius of 2.3 cm and a height of 1.4 m
- c) a cubic box with an edge of 1.2 cm



16. During a short interval of time the speed v in m/s of an automobile is given by $v = at^2 + bt^3$, where the time t is in seconds. Please determine the units of a and b

Vectors

17. Four vectors ($\vec{A}, \vec{B}, \vec{C}, \vec{D}$) all have the same magnitude. The angle between adjacent vectors is 45° as shown. Please choose (A – E) and justify the correct vector equation.

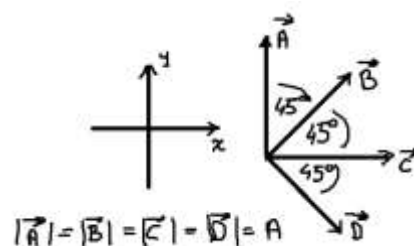


- A. $\vec{A} - \vec{B} - \vec{C} + \vec{D} = 0$
- B. $\vec{B} + \vec{D} - \sqrt{2}\vec{C} = 0$
- C. $\vec{A} + \vec{B} = \vec{B} + \vec{D}$
- D. $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$
- E. $(\vec{A} + \vec{C})/\sqrt{2} = -\vec{B}$

18. Four vectors ($\vec{A}, \vec{B}, \vec{C}, \vec{D}$) all have the same magnitude

A. The angle between adjacent vectors is 45° as shown. Please find the coordinates of each of the vectors and then find the coordinates of (express them by A):

- d) $\vec{A} - \vec{B} - \vec{C} + \vec{D}$
- e) $(\vec{A} + \vec{C})/\sqrt{2} + \vec{B}$



E

19. A vector of magnitude 20 is added to a vector of magnitude 25. Please give the range in which the magnitude of this sum can be.
20. A vector has a component of 10 m in the +x direction, a component of 10 m in the +y direction, and a component of 5 m in the +z direction. Please calculate the magnitude of this vector.

21. Two vectors $\vec{a} = 1\hat{i} + 3\hat{j} - 2\hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$ are given. Please find:

- a) length of each vector
- b) dot product $\vec{a} \cdot \vec{b}$
- c) angle between $\vec{a} - \vec{b}$ and $\vec{a} + \vec{b}$
- d) vector product $\vec{a} \times \vec{b}$

22. Let us assume that: $\vec{a} = 3\hat{i} + 3\hat{j} - 2\hat{k}$; $\vec{b} = -\hat{i} - 4\hat{j} + 2\hat{k}$; $\vec{c} = 2\hat{i} + 2\hat{j} + \hat{k}$
Please check if $\vec{a} \perp \vec{b}$ or $\vec{a} \parallel \vec{b}$ or $\vec{b} \perp \vec{c}$ and determine:

- a) $\vec{a} \cdot (\vec{b} \times \vec{c})$
- b) $\vec{a} \cdot (\vec{b} + \vec{c})$
- c) $\vec{a} \times (\vec{b} + \vec{c})$
- d) $\vec{a} \times (\vec{b} \times \vec{c})$

23. Please find (in case of vector products express your results by $\hat{i}, \hat{j}, \hat{k}$ and calculate their magnitudes):

a) $\hat{i} \cdot \hat{k}$

c) $\hat{j} \cdot (-\hat{j})$

e) $(-\hat{i}) \times (-\hat{j})$

b) $(-\hat{k}) \cdot (-\hat{j})$

d) $\hat{k} \times \hat{j}$

f) $\hat{k} \times (-\hat{j})$

24. There are two vectors $\vec{a}$ and $\vec{b}$ attached to the same point. The angle α between the vectors is acute. Please find this angle if increasing α by $\Delta\alpha = 20$ deg will result in:

- a) the vector product having the same magnitude,
- b) the dot product will be the opposite.



25. Suppose that: $\vec{a} = -3\hat{i} + m\hat{j} - n\hat{k}$; $\vec{b} = -2\hat{i} - 4(n - m)\hat{j} + 2(n + 1)\hat{k}$, m, n – real numbers. Please find n and m if:

- a) $\vec{a} \parallel \vec{b}$,
- b) $\vec{a} \perp \vec{b}$ and $n = 2m$.



26. The magnetic force $\vec{F}_L$ acting on charge q (so-called *Lorentz force*) moving in the magnetic field $\vec{B}$ (sometimes called *magnetic induction* $\vec{B}$ or *magnetic vector* $\vec{B}$) with velocity $\vec{v}$ can be described as:

$$\vec{F}_L = q \cdot (\vec{v} \times \vec{B})$$

Please:

- a) discuss the relation between the direction of $\vec{v}$ and $\vec{F}_L$ and between the direction of $\vec{B}$ and $\vec{F}_L$;
- b) determine $\vec{F}_L$ if $q = -2e = -3.2 \cdot 10^{-19}$ C ; $\vec{v} = [2, 1, 0] \cdot 10^5$ m/s ; $\vec{B} = [0.2 ; 1 ; 0.3]$ T and calculate the magnitude of $\vec{F}_L$
- c) find the angle between $\vec{v}$ and $\vec{B}$ (values of q , $\vec{B}$ $\vec{v}$ in part b)).