

Probability and Statistics

Set no. 4

random variables

1. The table shows the weight and height of 10 people.

person	A	B	C	D	E	F	G	H	I	J
Mass, kg	81	76	55	102	100	150	63	98	71	78
Height, cm	182	171	180	191	160	173	196	180	185	172

Please calculate the $BMI = M [kg] / (H [m])^2$ for each person and then find the probability distribution $p(x)$ of the random variable if the random variable is given by:

BMI range	< 20	[20,25)	[25,30)	[30,35)	[35,40]	>40
X(BMI)	1	2	3	4	5	6

Please graph $p(x)$ and calculate $E(X)$ and $D(X)$ for this distribution.

2. Cumulative distribution function (CDF), $F(x)$ of the random variable X is defined by the table below. Determine probability distribution (probability mass function).

X	$(-\infty, -2)$	$[-2, 3)$	$[3, 5)$	$[5, +\infty)$
$F(x)$	0	0.4	0.5	1

3. Students wrote an exam in physics. Let's assume that the number of obtained grades (5.0; 4.0; 3.0; 2.0) remains in the following ratio 1 : 3 : 4 : 2. The task is to find / determine:

- probability distribution $p(x)$ and its plot
- cumulative distribution function $F(x)$ and its plot
- probability $P(X \leq 3.5)$ relying on: (i) the probability distribution ; (ii) cumulative distribution function $F(x)$
- probability $P(3 \leq X < 4.5)$ relying on: (i) the probability distribution ; (ii) cumulative distribution function $F(x)$

4. The probability distribution $p(x)$ is given by:

x	-5	-3	0	2
p_i	0.1	0.4	0.3	?

Please find the value of ? and CDF for this distribution.

5. Let's assume that the probability distribution $p(x)$ is:

$$p(x) = \frac{20}{49x} \text{ where } x = 1, 2, 3, \dots, n.$$

Please find n ($n < 10$) and then $E(X)$.

6. Let's have random variable X and its probability mass function given by the table below. Determine: (a) expected value $E(X)$, (b) median $x_{0.5}$, (c) variance $D^2(X)$, (d) standard deviation $D(X)$.

x_i	-2	2	4
p_i	0.5	0.3	0.2

7. Please find $E(X)$ if the probability mass function is given as:

$$p(x) = \frac{2}{3^x}, \text{ where } x = 1, 2, 3, \dots$$